

# Quadratic Expressions and Equations - Mathematics Form 3 Notes

- [Perfect Square](#)
- [Completing the Square](#)
  - [Solutions of Quadratic Equations by Completing the Square Methods](#)
- [The Quadratic Formula](#)
- [Formation of Quadratic Equations](#)
- [Graphs of Quadratic Functions](#)
- [Graphical Solutions of Simultaneous Equations](#)
- [Past KCSE Questions on the Topic.](#)

## [Perfect Square](#)

- Expressions which can be factorized into two equal factors are called perfect squares.

## [Completing the Square](#)

- Any quadratic expression can be simplified and written in the form  $ax^2 + bx + c$  where  $a$ ,  $b$  and  $c$  are constant and  $a$  is not equal to zero. We use the expression  $(\frac{b}{2})^2 = C$  to make a perfect square
- We are first going to look for expression where coefficient of  $x = 1$

### Example

What must be added to  $x^2 + 10x$  to make it a perfect square?

### Solution

- Let the number to be added be a constant  $c$ .
- Then  $x^2 + 10x + c$  is a perfect square.
- Using  $(\frac{b}{2})^2$
- $(\frac{10}{2})^2 = c$
- $c = 25$  (25 must be added)

### Example

What must be added to  $x^2 + \_ + 36$  to make it a perfect square

### Solution

- Let the term to be added be  $bx$  where  $b$  is a constant
- Then  $x^2 + bx + 36$  is a perfect square.
- Using  $(\frac{b}{2})^2 = 36$
- $\frac{b}{2} = \sqrt{36}$
- $\frac{b}{2} = \pm 6$   $b = 12$  or  $-12$

We will now consider the situations where  $a \neq 1$  and not equal to zero eg

$$4x^2 - 12x + 9 = (2x - 3)^2$$

$$9x^2 - 6x + 1 = (3x - 1)^2$$

In the above you will notice that  $(\frac{b}{2})^2 = \frac{c}{a}$ . We use this expression to make perfect squares where  $a$  is not one and its not zero.

### Example

What must be added to  $25x^2 + \_ + 9$  to make it a perfect square?

### Solution

- Let the term to be added be  $bx$ .
- Then,  $25x^2 + bx + 9$  is a perfect square.
- Therefore  $(b/2)^2 = 25 \times 9$ .
- $(b/2)^2 = 225$
- $b/2 = \pm 15$
- so  $b = 30$  or  $-30$  The term to be added is thus  $30x$  or  $-30x$ .

### Example

What must be added to  $\_ - 40x + 25$  to make it a perfect square?

### Solution

- Let the term to be added be  $ax^2$
- Then  $ax^2 - 40x + 25$  is a perfect square.
- Using  $(b/2)^2 = ac$
- $(-40/2)^2 = 25a$
- $400 = 25a$
- $a = 16$  the term to be added is  $16x^2$

## Solutions of Quadratic Equations by Completing the Square Method

### Example

Solve  $x^2 + 5x + 1 = 0$  by completing the square.

### solution

$x^2 + 5x + 1 = 0$  Write original equation.

$x^2 + 5x = -1$  Write the left side in the form  $x^2 + bx$ .

$x^2 + 10x + (5/2)^2 = (5/2)^2 - 1$  Add  $(5/2)^2$  to both sides

$x^2 + 10x + 25/4 = 21/4$

$(x + 5/2) = 21/4$  Take square roots of each side and factorize the left side

$x + 5/2 = \pm \sqrt{21/4}$  Solve for  $x$ .

$= -5/2 \pm 4.583/2$  Simplify

$= 0.417/2$  or  $9.583/2$  Therefore  $x = -0.2085$  or  $4.792$

The method of completing the square enables us to solve quadratic equations which cannot be solved by factorization.

### Example

Solve  $2x^2 + 4x + 1 = 0$  by completing the square