Matrices and Transformation - Mathematics Form 4 Notes

Introduction

- A transformation change the shape, position or size of an object as discussed in book two.
- Pre – multiplication of any 2 x 1 column vector by a 2 x 2 matrix results in a 2 x 1 column vector

Example 1

$$\begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ -1 \end{bmatrix} = \begin{bmatrix} 17 \\ -9 \end{bmatrix}$$

If the vector $\begin{bmatrix} 7 \\ -1 \end{bmatrix}$ is thought of as a position vector that is to mean that it is representing the points with coordinates (7, -1) to the point (17, -9).

Note;

- The transformation matrix has an effect on each point of the plan. Let's make T a transformation matrix $T\begin{bmatrix} 7 \\ -1 \end{bmatrix}$ Then T maps points (x, y) onto image points x^1 ,y

$$T\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x^1 \\ y^1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}\begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} x^1 \\ y^1 \end{bmatrix}$$

$$= \begin{bmatrix} 3x+4y \\ -1x+2y \end{bmatrix}$$

Finding the Matrix of Transformation

- The objective is to find the matrix of given transformation.

Example 2

Find the matrix of transformation of triangle PQR with vertices P (1, 3) Q (3, 3) and R (2, 5).The vertices of the image of the triangles is P^1(1,-3) ,Q^1(3,-3) and R^1(2,-5).

Solution

Let the matrix of the transformation be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Equating the corresponding elements and solving simultaneously

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} P & Q & R \\ 1 & 3 & 2 \\ 3 & 3 & 5 \end{pmatrix} = \begin{pmatrix} P^1 & Q^1 & R^1 \\ 1 & 3 & 2 \\ -3 & -3 & -5 \end{pmatrix}$$

$$\begin{pmatrix} a+3b & 3a+3b & 2a+5b \\ c+3d & 3c+3d & 2c+5d \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \\ -3 & -3 & -5 \end{pmatrix}$$

a + 3b = 1
<u>3a + 3b = 3</u>
2a= 2
a = 1 and b = 0
c + 3d = -3
<u>3c + 3d = -3</u>
2c= 0
c = 0 and d = -1

Therefore the transformation matrix is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Example 3

A trapezium with vertices A (1 ,4) B(3,1) C (5,1) and D(7,4) is mapped onto a trapezium whose vertices are A^1(-4,1) ,B^1(-1 ,3) ,C^1(-1,5) ,D^1(-4 ,7).Describe the transformation and find its matrix

Solution

Let the matrix of the transformation be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Equating the corresponding elements we get;

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} A & B & C & D \\ 1 & 3 & 5 & 7 \\ 4 & 1 & 1 & 4 \end{pmatrix} = \begin{pmatrix} A^1 & B^1 & C^1 & D^1 \\ -4 & -1 & -1 & -4 \\ 1 & 3 & 5 & 7 \end{pmatrix}$$

a + 4b = - 4   c + 4d = 1
3a + b = -1   3c + d = 3

Solve the equations simulteneously
3a + 12b = - 12
<u>3a + b = - 1</u>
11b = -11
hence b =-1 or a = 0

3c + 12d = 3
<u>3c + d =3</u>
11d = 0
d = 0 c = 1